Written re-exam for the M. Sc. in Economics. Summer 2013

Economic Growth

Master's Course

August 14, 2013

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is: Problem 1: 40%, Problem 2: 45%, and Problem 3: 15%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a simple closed economy where two goods get produced: corn and new ideas. Time is continuous. The labor force (in this problem the same as population), L_t, grows at an exogenous constant rate $n \geq 0$, i.e., $L_t = L_0 e^{nt}$. People can work as farmers, L_{Yt}, or researchers, L_{At}, where $L_{Yt} + L_{At} = L_t$ and $L_{At} = s_A L_t$, where, for simplicity, s_A is assumed constant, $0 < s_A < 1$. The production functions are:

$$\begin{aligned} Y_t &= A_t^{\sigma} J^{\alpha} L_{Yt}^{1-\alpha}, \qquad \sigma > 0, 0 < \alpha < 1, & (*) \\ \dot{A}_t &= \mu A_t^{\varphi} L_{At}, \qquad \mu > 0, \varphi < 1, & (**) \end{aligned}$$

where Y_t is corn output, J is the amount of land (fixed), and A_t the level of technical knowledge. Let the growth rate of a variable $x > 0$ at a given point in time be denoted g_x (not necessarily a constant). Let $y \equiv Y/L$.

a) Express g_y in terms of g_A and n.

b) Sign g_A.

c) Show that $g_A \to n/(1-\varphi)$ for $t \to \infty$. *Hint:* The easiest approach is to calculate the growth rate of g_A on the basis of (**); by reordering you get an expression of the form $\dot{x} = (a - bx)x$ where $a \geq 0$ and $b > 0$; now use the principle that when $x > 0$, we have $\dot{x} \gtreqless 0$ for $x \lesseqgtr a/b$, respectively.

d) Let the long-run value of g_y be denoted g_y^*. Find g_y^*.

e) Let $n > 0$. Under three alternative conditions involving the values of α, σ, and n, three alternative cases, $g_y^* > 0, g_y^* = 0$, and $g_y^* < 0$, are possible. Show this.

f) In the case $g_y^* > 0$ show that $\partial g_y^* / \partial n > 0$. What is the economic intuition behind this result?

g) What is the economic intuition behind that the alternative case, $g_y^* < 0$, is also possible in the model?

h) Would this alternative case be possible if the marginal productivity of labor in corn production were a positive constant? Why or why not?

Problem 2 Consider a closed economy with two production sectors, manufacturing and R&D. Time is continuous. At the aggregate level we have:

$$\begin{aligned} Y_t &= T_t K_t^{\alpha} (\bar{h}_t L_{Yt})^{1-\alpha}, \qquad 0 < \alpha < 1, & (1) \\ \dot{K}_t &= Y_t - c_t N_t - \delta K_t, \qquad \delta \geq 0, & (2) \\ T_t &= A_t^{\sigma}, \qquad \sigma > 0, & (3) \\ \dot{A}_t &= \eta A_t^{\varphi} \bar{h}_t L_{At}, \qquad \eta > 0, \varphi < 1, & (4) \\ L_{Yt} + L_{At} &= L_t, & (5) \end{aligned}$$

where Y_t is manufacturing output, K_t is capital input, $\bar{h}_t$ is average human capital in the labor force, c_t is per capita consumption, and N_t is population at time t; otherwise notation corresponds to that in Problem 1.

Assume that all variables in the model are positive and remain so.

a) On the basis of (1), find the TFP growth rate in the manufacturing sector, ignoring the other equations.

b) Regarding (4), interpret the case $0 < \varphi < 1$ versus the case $\varphi < 0$.

c) Presupposing $g_A > 0$, express the growth rate of g_A in terms of g_A, $g_{\bar{h}}$, and g_{L_A}. *Hint:* You may start by expressing g_A in terms of A, $\bar{h}$, and L_A; then take logs and then time derivatives.

Let the time unit be one year. Suppose an individual "born" at time v (v for "vintage") spends the first S years of life in school and then enters the labor market with a human capital level which at time $t \geq v + S$ is $h(S)$, where $h' > 0$. We ignore the role of teachers and schooling equipment. We assume that life expectancy is constant over time and that S is the same for all individuals independently of time of birth. After leaving school, individuals works full-time until death. We assume that the population grows at a constant rate $n > 0$:

$$N_t = N_0 e^{nt}. \tag{6}$$

Then, with a stationary age distribution in society,

$$L_t = (1 - \beta)N_t, \tag{7}$$

where β is the constant fraction of this population under education (β will be an increasing function of S).

From now on assume that (i) the economy has balanced growth, defined as a path along which g_Y, g_C, g_K, g_A, $g_{\bar{h}}$, g_{L_A}, and g_{L_Y} are constant; (ii) $Y - cN > 0$ for all t.

d) Can we be sure that $g_{L_Y} = n$ along the balanced growth path (BGP)? Why or why not? *Hint:* First find g_L; then think about what happens if $g_{L_Y} \neq n$ but g_{L_Y} still constant?

e) From a certain general proposition we can be sure that along a BGP, $g_Y = g_K$. What does this proposition say?

f) Defining $y \equiv Y/L$, on the basis of your result at a), find g_y expressed in terms of g_T and g_k, where $k \equiv K/L_Y$.

g) Applying also (3), show that the expression for g_y can be reduced to one in terms of only g_A.

h) Compare your result at g) with that at f). On this basis evaluate the statement: "The TFP growth rate measures the contribution to productivity growth from growth in technology".

i) As g_A itself is endogenous in the complete model, you are now asked to derive a full determination of, first, g_A, and then g_y under balanced growth. Comment.

Problem 3 *Short questions*

a) It is sometimes argued that results like the Arrow result, $g_y = \lambda n/(1-\lambda)$ (standard notation), are from an empirical point of view falsified by the fact that cross-country growth regressions do not tend to indicate a positive correlation between per capita economic growth and population growth. Evaluate this argument.

b) Consider a set of countries, $j = 1, 2, ..., N$. Country j has the aggregate production function

$$Y_{jt} = F(K_{jt}, A_{jt}L_{jt}),$$

where F is neoclassical and has CRS (standard notation). The technology level A_{jt} evolves according to $A_{jt} = A_{j0}e^{gt}$, where A_{j0} differs widely across the countries. The positive constant g as well as the function F and the capital depreciation rate are, however, the same across the countries. Assume that (i) the countries trade in a fully integrated world market for goods and financial capital; (ii) they face a constant real interest rate $r > 0$ in this market; and (iii) there is perfect competition in all markets. "In this setup there will be a strong economic incentive for workers to migrate." True or false? Explain why.

c) Our syllabus describes two main versions of the expanding input variety model and two sub-versions of one of these. The key difference is in the specification of invention production function. Briefly characterize the difference between the three specifications.

—